4.6

Identity Theft

Exploring Polynomial Identities

LEARNING GOALS

In this lesson, you will:

- Use polynomial identities to rewrite numeric expressions.
- Use polynomial identities to generate Pythagorean triples.
- Identify patterns in numbers generated from polynomial identities.
- Prove statements involving polynomials.

KEY TERM

- Euclid's Formula

Have you or someone you know ever been the victim of identity theft? With more and more tasks being performed through the use of technology, identity theft is a growing problem throughout the world. Identity theft occurs when someone steals another person's name, or social security number in hopes of accessing that person's money or to make fraudulent purchases.

There are many different ways a person can steal another person's identity. Just a few of these methods are:

- rummaging through a person's trash to obtain personal information and bank statements,
- computer hacking to gain access to personal data,
- pickpocketing to acquire credit cards and personal identification such as passports or drivers' licenses,
- browsing social networking sites to obtain personal details and photographs.

How important is it to you to secure your identity? What actions would you take to ensure that your identity is not stolen?

PROBLEM 1 Check Your Calculator at the Door

You have learned about many different equivalent polynomial relationships. These relationships are also referred to as polynomial identities.

Some of the polynomial identities that you have used so far are shown.

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a + b)(a - b)$
- $(a + b)^3 = (a + b)(a^2 + 2ab + b^2)$
- $(a - b)^3 = (a - b)(a^2 - 2ab + b^2)$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Polynomial identities can help you perform calculations. For instance, consider the expression 46^2. Most people cannot calculate this value without the use of a calculator. However, you can use a polynomial identity to write an equivalent expression that is less difficult to calculate.

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You can use the polynomial identity $(a + b)^2 = a^2 + 2ab + b^2$ to calculate 46^2.

$46^2 = (40 + 6)^2$	Write 46 as the sum of 40 and 6.
$= 40^2 + 2(40)(6) + 6^2$	Apply the polynomial identity $(a + b)^2 = a^2 + 2ab + b^2$.
$= 1600 + 2(40)(6) + 36$	Apply exponents.
$= 1600 + 480 + 36$	Perform multiplication.
$= 2116$	Perform addition.

The value of 46^2 is 2116.

1. Calculate 46^2 in a different way by writing 46 as the difference of two integers squared.

2. Use polynomial identities and number properties to perform each calculation. Show your work.

a. 112^2

b. 27^3

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c. 55^3

PROBLEM 2 It's Triplets!

Remember that a Pythagorean triple is a set of three positive integers, a, b, and c, such that $a^2 + b^2 = c^2$.

1. Determine whether each set of numbers is a Pythagorean triple. Explain your reasoning.

   a. 4, 5, 9

   b. 0.4, 0.5, 0.3

   c. 89, 80, 39

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You have just determined whether three positive numbers make up a Pythagorean triple, but suppose that you wanted to *generate* integers that are Pythagorean triples.

2. Describe a process you could use to calculate integers that are Pythagorean triples.

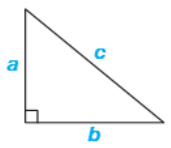
There is an efficient method to generate Pythagorean triples that involves a polynomial identity called *Euclid's Formula*.

Euclid's Formula is a formula used to generate Pythagorean triples given any two positive integers. Given positive integers r and s, where $r > s$, Euclid's Formula is shown.

$$(r^2 + s^2)^2 = (r^2 - s^2)^2 + (2rs)^2$$

The expressions in Euclid's Formula represent the side lengths of a right triangle, a, b, and c, as shown.

$$c^2 = a^2 + b^2$$
$$(r^2 + s^2)^2 = (r^2 - s^2)^2 + (2rs)^2$$



You can verify Euclid's Formula by transforming the right side of the equation to show that it is equal to the left side.

Given positive integers a and b, where $a > b$.

$(a^2 + b^2)^2 \stackrel{?}{=} (a^2 - b^2)^2 + (2ab)^2$	Apply Euclid's Formula.
$\stackrel{?}{=} a^4 - 2a^2b^2 + b^4 + (2ab)^2$	Square the binomial.
$\stackrel{?}{=} a^4 - 2a^2b^2 + b^4 + 4a^2b^2$	Apply Product to a Power Rule.
$\stackrel{?}{=} a^4 - 2a^2b^2 + 4a^2b^2 + b^4$	Apply the Associative Property of Addition.
$\stackrel{?}{=} a^4 + 2a^2b^2 + b^4$	Combine like terms.
$= (a^2 + b^2)^2$	Factor perfect square trinomial.

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3. Use Euclid's Formula to generate a Pythagorean triple.

   a. Choose two integers and use them to generate a Pythagorean triple. Explain your choice in integers.

   b. Compare your Pythagorean triple to others in your class. Did everyone get the same triple?

4. Generate a Pythagorean triple using each pair of given numbers and Euclid's Formula.

   a. 4 and 7

   b. 11 and 5

   c. 15 and 20

5. Did any of the Pythagorean triples you generated have a common factor? If so, identify them, and explain why you think this happened.

6. The integers 5, 12, 13 make up a fairly well-known Pythagorean triple. What two integers generate this triple? Show your work.

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PROBLEM 3 Is This Your Special Number?

After learning that Euclid's Formula generates numbers that are Pythagorean triples, Danielle and Mike wonder what other formulas they could use to generate interesting patterns. Each came up with their own sets of numbers.

Danielle named her numbers the "Danielle numbers." She defined them as shown.

> The Danielle numbers are any numbers that can be generated using the formula $a^2 + b^2$, where a and b are positive integers and $a > b$.

Following suit, Mike named his numbers the "Mike numbers," and he defined his numbers as shown.

> The Mike numbers are any numbers that can be generated using the formula $a^2 - b^2$, where a and b are positive integers and $a > b$.

1. Complete each table to determine the first few Danielle numbers, and the first few Mike numbers. Shade the corresponding cell if a is not greater than b.

Danielle Numbers: $a^2 + b^2$

		b				
		1	2	3	4	5
a	1					
	2					
	3					
	4					
	5					

Mike Numbers: $a^2 - b^2$

		b				
		1	2	3	4	5
a	1					
	2					
	3					
	4					
	5					

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2. Describe any and all patterns you see in each table in Question 1.

3. Determine whether each number is a Danielle number, a Mike number, both, or neither. Explain your reasoning.

a. 13

4

b. 3

c. 2

After hearing about Danielle and Mike's numbers, Dave and Sandy decide to create their own numbers as well. Their definitions are shown.

The Dave numbers are any numbers that can be generated using the formula $a^3 + b^3$, where a and b are positive integers and $a > b$.

The Sandy numbers are any numbers that can be generated using the formula $a^3 - b^3$, where a and b are positive integers and $a > b$.

4. Complete the tables to determine the first few Dave numbers, and the first few Sandy numbers. Shade the corresponding cell if a is not greater than b.

Dave Numbers: $a^3 + b^3$

		b				
		1	2	3	4	5
a	1					
	2					
	3					
	4					
	5					

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Sandy Numbers: $a^3 - b^3$

		b				
		1	2	3	4	5
a	1					
	2					
	3					
	4					
	5					

5. Describe any and all patterns you see in each table in Question 4.

6. Determine whether each number is a Dave number, a Sandy number, both, or neither. Explain your reasoning.

a. 35

4

b. 5

7. Write a rule that defines your own set of numbers. What interesting patterns do you see with your numbers?

PROBLEM 4 Okay, Now Prove It!

Verify each algebraic statement by transforming one side of the equation to show that it is equivalent to the other side of the equation.

1. $v^6 - w^6 = (v^2 - w^2)(v^2 - vw + w^2)(v^2 + vw + w^2)$

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2. $(p^4 + q^4)^2 = (p^4 - q^4)^2 + (2p^2q^2)^2$

3. $m^9 + n^9 = (m + n)(m^2 - mn + n^2)(m^6 - m^3n^3 + n^6)$

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Be prepared to share your solutions and methods.